A new method for determination of magnetization direction
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Summary
The interpretation and characterization of magnetic anomalies is reliant upon the knowledge of the total magnetization direction, usually assumed to be caused solely or primarily by induced magnetization. The presence of strong remanent magnetization can adversely affect the interpretation of the magnetic data and lead to erroneous size or shape. Therefore, it is imperative to know the total magnetization direction. We propose a method that is based upon the optimal correlation between two tools in magnetic data interpretation: the vertical gradient and the total gradient of the RTP field. In this paper we discuss the need for such a method, outline the theory and numerical implementation, and test it on both synthetic and field data sets.

Introduction
For proper interpretation of a magnetic data set or anomaly, it is imperative that the true total magnetization direction is known. In most cases the induced magnetization in the ambient field direction is assumed to be the only or primary component of the total magnetization and its direction is assumed to be known. This can lead to false confidence in interpretation of magnetic data. However, in many cases remanence is present and it is strong enough to affect the true magnetization direction.

Remanent magnetization is usually defined as the magnetization embedded in the mineral composition of a rock due to the direction of the Earth’s field when the rock was formed. Remanence can also be caused by chemical reactions in the rock after formation, fluctuation in temperature above and below the Curie point, or long-term exposure to an external magnetic field. Most remanent magnetization that affect exploration problem is due to thermal remanent magnetization.

The proper determination of the total magnetization direction or the effects of remanence on an anomaly is a problem that has been examined numerous times prior to this work. Zietz and Anderson (1967) looked at the relationship between position and intensity of the maximum and minimum produced by a sample body. Roest and Pilkington (1993) correlated the amplitude of the total gradient of the magnetic field and the horizontal gradient of the pseudogravity. Haney and Li (2002) developed a wavelet-based method for 2D data sets. Phillips (2003) developed a method based upon the integral relationships derived by Helbig (1962). These are just a few examples of work that has been done in this area and each of the methods carry with them strengths and weaknesses. As an example, Roest and Pilkington (1993) state that their method encounters difficulties when the magnetic body is dipping. Our attempt to improve upon their method is the initial basis from which we have derived the method presented here.

In order to properly reduce data to the pole, either the total magnetization direction or an accurate approximation must be known. The question then becomes: how does one define data that has been properly reduced to the pole? The search for this criteria or quantity within the data that defines a vertical magnetic anomaly with a vertical total magnetization direction is the basis for our method. This paper focuses on developing such a criterion and verifying it with synthetic and field data sets that are produced with different source configurations.

Theory and Implementation
As discussed previously Roest and Pilkington (1993) examined the correlation in structure between the total gradient and pseudo-gravity computed numerically from magnetic data. Their reasoning is that both quantities are symmetric if the correct direction is used in the calculation of pseudo-gravity. The difficulty with this method is that the two quantities decay at different rates with respect to the depth of the anomalous body. For that reason, the method does not work well with a dipping source.

Our method involves the correlation of the vertical gradient of the magnetic anomaly reduced to the pole and the total gradient of the same field. The total gradient of correct RTP field should be most symmetric, since it is the envelope of the vertical derivatives of magnetic anomalies. This provides us with a basis for comparison to gauge the symmetry of the RTP data. The cross-correlation between the two quantities should achieve a maximum when the correct magnetization direction is used for RTP calculation. The estimation process becomes one of searching for the inclination and declination that maximize the cross-correlation.

We note that both quantities decay at the same rate with depth. For instance, they are proportional to $1/z^2$ for a spherical source. The relative contributions to both quantities from sources at different depths are similar,
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The shapes of the total gradient and vertical gradient are affected in the similar manner by source depth. For this reason, our new approach seems to work well with dipping bodies.

Assuming we have total-field anomalies as our data, the first step in the estimation process is to perform a half reduction to the pole using the known direction of the inducing field. This converts the data to vertical anomalies and leaves only the effect of magnetization direction to be considered. To perform half reduction to the pole we transformed the data set to the Fourier domain and applied a simple multiplication operator to the data set,

$$\Psi_{\text{HRTP}} = \frac{\sqrt{\omega_x^2 + \omega_y^2}}{(K \cdot \hat{B}_0)}$$  
(1)

where $\omega_x$ and $\omega_y$ are respectively the wave numbers in the x- and y-directions, $K = (i \omega_x, i \omega_y, \sqrt{\omega_x^2 + \omega_y^2})$, and $\hat{B}_0$ is the unit directional vector of inducing field.

With the derived vertical anomalies, we can now proceed with the estimation. The criterion is the cross-correlation between the vertical gradient and total gradient of the "RTP" field. It is important to note that this quantity is not the RTP field in the strict sense of the word. Instead, it is a set of numerical values obtained by applying another half RTP operator with a trial magnetization direction. This operator is similar to that in eq.(1) with the directional vector replaced by $\hat{M}$. Let the result of the second HRTP operation be denoted as $R$, its vertical derivative as $v$, and total gradient as $g$:

$$v = \frac{\partial R}{\partial z}$$
$$g = \sqrt{(\frac{\partial R}{\partial x})^2 + (\frac{\partial R}{\partial y})^2 + (\frac{\partial R}{\partial z})^2}$$  
(2)

Both $v$ and $g$ are functions of assumed magnetization direction, $\hat{M}_n, n = 1, \ldots, N$.

Once both the vertical and total gradient are computed, the cross-correlation coefficients can be evaluated according to the following formula:

$$C = \frac{\sum (v_j - \bar{v})(g_j - \bar{g})}{\sqrt{\sum (v_j - \bar{v})^2 \sum (g_j - \bar{g})^2}}$$

(3)

where $j$ is the index of each quantity within the data set, and $\bar{v}$ and $\bar{g}$ are respectively the mean of each quantity.

The search for maximum correlation coefficient can be carried out in two ways. First, we can fix the declination and only search for the inclination of magnetization. At intermediate to low latitudes, we may be able to determine the declination based upon the power spectrum of the half reduced-to-pole data. At these latitudes, there is a pronounced depression in the power spectrum along the direction perpendicular to the declination direction. We then need only a line search to find the inclination.

In general, however, neither inclination nor declination is known and we must simultaneously search for both. This involves a bi-variant optimization. We can either employ a downhill search algorithm or compute the correlation coefficients for a grid of trial directions and find the maximum. The former approach is more efficient and appropriate for large data sets.

Figure 1. The magnetic data set computed from our synthetic model.

Example

The synthetic model created for testing our algorithm has three separate prismatic source bodies at varying depths. This model is chosen to show that our method can handle bodies at different depths and/or dipping bodies. The prisms have varying volumes but each has a magnetic susceptibility of 0.5. The anomalous bodies were each given a total magnetization that is a different direction from the inducing field. The ambient field for this model has an inclination of 70° and a declination of 30°. The magnetization has an inclination of 35° and a declination of 20°. Figure 1 shows the magnetic field produced from the three-body model.

Figures 2 and 3 show, respectively, the vertical gradient and the total gradient of true RTP anomaly obtained by specifying the correct magnetization direction. These are
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shown to exemplify how closely correlated the two quantities are given the true magnetization direction. As discussed above, the two quantities shown in Figures 2 and 3 should have the highest correlation coefficient.

![Figure 2](image1.png)

**Figure 2.** The vertical gradient computed from our data set after reduction to the pole.

![Figure 3](image2.png)

**Figure 3.** The total gradient computed from our data set after reduction to the pole.

We applied the first variant of our method to the example model: we assume that the declination is known based upon the power spectrum. The result of this process is shown in Figure 4, which is a polar plot showing the correlation coefficient as a function of trial inclination. The curve achieves maximum at 34°, which is in good agreement with the true inclination of 35°. Thus the method has proved successful in this case.

Now we remove the assumption of known declination and use the full search method to determine both inclination and declination. The results of our method are shown in Figure 5, which is a contour plot of the correlation coefficient as a function of trial inclination and declination. The result is equally encouraging. The correlation coefficient peaks at an inclination of 35° and a declination of 20°, which is same as our true values.

![Figure 4](image3.png)

**Figure 4.** A polar plot of the inclination values derived from our method with a fixed declination at ranges from -90° to 90° excluding low latitudes. The true inclination is 35°. The solid lines indicate positive correlation and the dashed lines indicate negative correlation.

![Figure 5](image4.png)

**Figure 5.** The correlation coefficients from our algorithm plotted over a range of inclination and declination values. The true inclination and declination are 35° and 20°, respectively.

**Discussions**

Knowledge of the true magnetization direction of an anomalous body is crucial when interpreting magnetic data.
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Without this knowledge the data cannot be properly reduced to the pole, nor can the size and shape of the source be recovered correctly through inversion. We have developed a new approach to help estimate the magnetization direction. The method is based upon the cross-correlation between the vertical gradient and the total gradient of RTP field obtained through a set of trial direction. The method is effective at determining the inclination of the total magnetization direction for general source geometry including dipping bodies.

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References


