Enhancement of depth estimation techniques with amplitude analysis
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SUMMARY

Depth estimation has been used widely as a tool for rapid interpretation of large-scale potential-field data in applications such as mapping basement relief. Nearly all of these techniques rely on the analysis of the local shape of the anomalous field in determining the depth and location of the subsurface sources. These methods focus on the phase information at the expense of the amplitude of the data. Consequently, these methods often produce a large number of solutions and interpretation of the result is difficult. We develop a method for enhancing these techniques by incorporating amplitude information back into the depth estimation process. The method statistically identifies significant source solutions from the estimation based on their relative source strengths, and discards false solutions due to noise and spray effects. The result is a subset of solutions that is more amenable for direct interpretation. We illustrate this new approach by applying it to the solution of the Euler and extended Euler deconvolutions. We demonstrate the improvement using magnetic data from the Bishop model, and present a field dataset from petroleum exploration.

INTRODUCTION

Depth estimation is a class of well-known techniques for finding the location and depth of potential-field sources. Traditionally, it has primarily been used to find depth to magnetic basement for petroleum exploration problems. There has been much published on the different techniques for depth estimation. These include the Naudy method (Naudy, 1971), Werner deconvolution (Werner, 1953), Euler deconvolution methods (Reid et al., 1990), the continuous wavelet method (Moreau et al., 1999), and the source parameter imaging methods (Thurstorn and Smith, 1997). A good summary of these methods is given by Li (2003). The methods can work well when the basic assumptions are met; however, they can be severely affected by any noise in the dataset and the choice of window size that may capture a partial anomaly or multiple anomalies.

To deal with these problems, much work has been done in several different aspects of data preparation such as enhancing the signal-to-noise ratio (SNR) by examining the derivatives of the field (e.g. Florio et al., 2006; Silva and Barbosa, 2003) or by combining different depth estimation methods (Salem and Ravat, 2003). These approaches focused upon the ability to more accurately calculate source location from the observed decay of the field from a single window. Although the solutions in general are improved, there may still be many that are manifestations of the noise within the data set or partial anomalies. Others focused on post-processing to improve the interpretability of the solutions. For example, Mikhailov et al. (2003) used artificial intelligence to better cluster the vast amount of solutions. Despite these efforts, depth estimation methods have had limited success. Part of the reason is that these methods still produce large numbers of depth solutions and, collectively, they are difficult to interpret.

To illustrate our methodology, we will use the popular approach of Euler deconvolution (Thompson, 1982) and extended Euler deconvolution (Mushayandebvu et al., 1999; Nabighian and Hansen, 2001). These two methods examine the shape of the magnetic field within a window and calculate three-dimensional source locations based on a structural index (SI). The structural index describes the rate the field decays based on source geometry and is specified based on expected target geometry in standard Euler deconvolution. For example, it is commonly agreed in the potential-field community that the SI should be between 0 and 0.5 in order to find basement depths. The extended Euler deconvolution has the option of solving for the SI. In both methods, problems develop and source locations can be inaccurate when noise or multiple sources are present within the window or the size of the window is too small to capture the anomaly shape. In the following, will discuss how these solutions can be directly related to the amplitude of the magnetic anomaly and how to properly discard them. Though we show a synthetic example using extended Euler deconvolution, our basic approach is general and applicable to any depth estimation method.

EULER DEPTH ESTIMATION

Euler deconvolution was originally developed in exploration geophysics for rapidly estimating the location and depth to magnetic or gravity sources. It is based on the fact that the potential field produced by many simple sources obeys Euler’s homogeneity equation (Hood, 1965). If a given component of the magnetic anomalous field ΔT (x, y, z) satisfies:

\[ ΔT (tx, ty, tz) = t^n ΔT (x, y, z) \] (1)

where \( n \) is the degree of homogeneity, then by differentiating Eq. (1) with respect to \( t \), it can be shown that

\[ x \frac{δΔT}{δx} + y \frac{δΔT}{δy} + z \frac{δΔT}{δz} = nΔT, \] (2)

where \( x, y, \) and \( z \) are the coordinates of field observation points and the source is assumed to be at the origin. Equation (2) is known as Euler’s homogeneity equation (Hood, 1965) or simply Euler’s equation. The degree of homogeneity is source dependent and characterizes how fast the field decreases as a function of distance to the source. For example, the total-field anomaly produced by a dipolar source decreases as inverse distance cubed and the corresponding degree of homogeneity is \( n = -3 \); a cylinder has \( n = -2 \); a dyke has \( n = -1 \); and a contact has \( n = 0 \). Typical Euler solutions for depth estimation use 0, 0.5, or 1.
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Equation (2) can be used to estimate the source depth from magnetic data in 3D, which gives rise to the method of Euler deconvolution (e.g., Reid et al., 1990). Since the potential field is inversely proportional to the distance raised to some power, the degree of homogeneity is non-positive. The negative of the degree of homogeneity is defined as the structural index (SI) and will be denoted as N. Nabighian and Hansen (2001) extended the work by Mushayandebvu et al. (1999) and showed that the Euler equation also holds true for the two components of the 3D Hilbert transforms of the field:

$$\sum_{x_i} (x_i - x_{io}) \frac{\partial H_x[\Delta T]}{\partial x_i} = -NH_x[\Delta T]$$

$$\sum_{x_i} (x_i - x_{io}) \frac{\partial H_y[\Delta T]}{\partial x_i} = -NH_y[\Delta T],$$

where $H_x[\Delta T]$ and $H_y[\Delta T]$ denote respectively the x- and y-component of the 3D Hilbert transform (Nabighian, 1984) applied to the field, $\Delta T$.

The beauty of these equations is that they seek to match the shape of the magnetic anomaly (or their Hilbert transforms) without worrying about the actual magnitude of the magnetic field. This is advantageous in theory since the Euler deconvolution, or any depth estimation, is a sequence of independent parametric inversions based on simplistic sources such as contacts or dykes and, consequently, the method is not concerned with reproducing the magnitude of the anomaly. For this reason, these methods can also deal with the unknown magnetization direction such as when remanence is present.

The drawback of these methods also stems from this inability. More specifically, geologically meaningful data are produced by complex sources that are not consistent with the simple parametric model and these data also contain certain amount of noise. These two aspects lead to the presence of false solutions in depth estimation. These solutions (structural index and source location) may be consistent with the local shape of the magnetic data within windows but they are in general not consistent with the magnitude of the anomaly. The result is a solution map with many source locations that are due to processing artifacts, cultural noise, or the partial capture of an anomaly, also called spray (Mushayandebvu et al., 2001; FitzGerald et al., 2004), which is of no interest in the final basement interpretation. It follows that identifying these false solutions is a logical direction to improving Euler results. We develop one approach in this direction by bringing back the amplitude information in the post-processing.

AMPLITUDE ANALYSIS

Published work (e.g., Reid et al., 1990; Mushayandebvu et al., 1999; FitzGerald et al., 2004) have illustrated the concept of the Euler deconvolution algorithm and its potential utility in first-pass quantitative interpretation of large-scale data sets. In application, however, common data noise and near-surface geological noise will strongly influence the performance of any estimation algorithm and lead to false solutions. In addition, a partial anomaly in any sliding window used for Euler deconvolution will degrade the solution and lead to strings of false solutions that are related to an authentic source location but is offset from it. This is the well-known spray effect. The cause of these false solutions lies in the fact that the extended Euler deconvolution solves for the source location and structural indices based on the decay of the field within the window regardless of amplitude. The rate of field decay is quantified through the shape of magnetic anomaly alone. Spatial variations in data due to the presence of noise or the partial capture of an anomaly lead to false solutions. Thus, complementary information from the amplitudes of magnetic anomalies is neglected in current depth estimation methods. We examine these issues here and develop a statistical method for identifying false Euler solutions by re-incorporating amplitude information into the final analysis.

In general, we have observed that the source strengths of magnetic anomalies are weaker for surface, spray, and small-scale geologic anomalies creating noise for the desired basement structure. Robust solutions associated with target geologic features tend to have relatively strong sources. It follows that knowledge of the source strength should provide additional information for us to distinguish between the strong magnetic basement and other sources. The quantification of source strength is, of course, based on the corresponding magnitude of magnetic anomaly. In theory, one can perform a least squares fit of data in a window for each depth solution to estimate the corresponding source strength. Doing so, however, will incur a prohibitive computational cost. Typical Euler deconvolution can produce in the order of millions of solutions. Alternatively, we choose to rely on the amplitude of anomalous field vector (henceforth referred to as amplitude data) computed from the total-field data as a proxy to the source strength.

We define the amplitude, $A$, as the magnitude of the anomalous field vector, $\vec{B}_a$:

$$A = |\vec{B}_a| = \sqrt{B_{ax}^2 + B_{ay}^2 + B_{az}^2},$$

where $B_{ax}$, $B_{ay}$, and $B_{az}$ are respectively the three orthogonal components of the anomalous field, $\vec{B}_a$. The measured total-field anomaly is the projection of $\vec{B}_a$ onto the inducing field direction. The amplitude data is an approximate envelope of total-field anomaly over all possible magnetization directions (Nabighian, 1972; Stavrev and Gerovska, 2000; Shearer, 2005) and the peak is centered nearly directly above the corresponding source in three dimensions. Furthermore, the amplitude peak value is proportional to source strength and inversely proportional to the depth raised to the power of structural index, $N$:

$$|\mathbf{m}| \propto h^NA(x_o, y_o),$$

where $|\mathbf{m}|$ is the magnitude of the source such as magnetization of a dyke or a dipole moment of compact source, $h$ is the depth to the source, and $A(x_o, y_o)$ is the magnetic amplitude data at the location $(x_o, y_o)$ of the depth-estimation solution. Consequently, we can obtain a relative measure of source strengths by simply examining the peak amplitude of each anomaly detected by Euler deconvolution. Thus, we define the relative
source strength, \( s \), as

\[
    s = h^N A(x_o, y_o),
\]

(6)

Scaling the amplitude data by the rate of decay using the structural index either calculated by extended Euler deconvolution, or chosen for traditional Euler deconvolution, is equivalent to removing the decay of the field with distance.

Generating these relative source strengths is straightforward and computationally inexpensive. It requires three linear transformations of the total-field anomaly to obtain the three orthogonal components in Eq. (4). This step is accomplished in wavenumber domain (Pedersen, 1978; Blakely, 1996). Scaling the amplitude data at each detected anomaly location according to Eq. (6) then yields relative source strengths for all anomalies from the Euler solutions.

It is observed that the Euler solutions predominately have rather weak sources and only a small percentage of them are strong and deviate from the majority. Closer inspection of the location of these anomalies show that the strong sources are indeed generally associated with basement features and the weaker ones are scattered in the portion of the data map where the signal-to-noise ratio (SNR) is low or belong to spraying stringers. Anomalies associated with spray are offset from the true source and thus have a weaker source strength than those directly over the source.

The problem of distinguishing between basement feature anomalies and false Euler solutions now becomes one of examining the difference in amplitude between these two groups of sources. This is accomplished by examining the histogram of log source strength. We discard the Euler solutions whose relative source strengths are less than the mean plus one standard deviation of all solutions. Solutions below this empirical threshold are considered to be from spray effect (as the majority of solutions are) and in most cases would not be considered viable magnetic basement features. This process windows most of the Euler solutions and identifies remaining ones as having significant strengths. The use of multiple window sizes ensures the best results and avoids bias towards a specific window size.

**SYNTHETIC EXAMPLE**

To illustrate the amplitude analysis as an effective post-processing criterion, we apply it to the solutions from Bishop data (Flanagan, 2008) obtained from multiple window sizes and extended Euler deconvolution. The calculated data are shown in Figure 1. As an example, we first performed extended Euler deconvolution on the data with a window size of 11 points (2200 m) (Figure 2) and the resulting structural indices range from 0.0 to 1.0. It is important to have a range of structural indices because the error in the extended Euler deconvolution algorithm is propagated to both the source location and structural index (i.e. the unknowns of the equation). This range enables us to capture all anomalies associated with geologic contacts. There are many solutions close to zero depth. This would not be a plausible result in order to map the basement reliably. We then perform extended Euler deconvolution using a sequence of window sizes ranging from 3 to 25 points. The reasoning is to minimize the effect of window size on the solutions. Again, we examine only the structural indices from 0 to 1 and calculate relative source strengths. The histogram of the Euler solutions is shown in Figure 3. We discard all solutions with a relative source strength below one standard deviation from the mean. The cut-off retains a small number of solutions. The results show that by using the amplitude, we are able to identify and keep only the solutions that directly correlate with major structural or susceptibility changes in the basement while discarding a large number of false solutions. The extended Euler results are shown in Figure 4 with the susceptibility in color fill, the basement relief in contours, and the extended Euler results after amplitude analysis in white dots. The results are either dominated by the relief, or by the susceptibility contrast. The influence of these two factors depends on the relative strength of their signals in the data. For the solutions dominated by the susceptibility, the depths do not agree well with the basement relief. The remaining results have a much better match with the depth to basement. With post-Euler amplitude analysis, we have selected a smaller number of solutions and these solutions are a direct effect of the basement. It is insightful to note that there are 124,210 solutions for a single window of 2200 by 2200 meters. The number increases dramatically to 1,635,263 when all solutions from twelve different window sizes are considered. However, after amplitude analysis of these solutions, only 25,263 solutions remain. These fewer solutions allow an interpreter to draw conclusions about the data much more easily. The solutions that were left also strongly correlated with either the basement relief features or the contacts of the differing susceptibilities. The amount of solutions after amplitude analysis would allow for parametric inversions that could be used with other techniques in order to synthesize a better quantitative understanding of basement relief in the area.
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Figure 2: Extended Euler solutions using a window size of 11 points (2200 m). The structural indices range from 0.0 to 1.0 and the depths from 0 to 6000 meters. There are 124,210 solutions with this window size.

CONCLUSIONS

We have developed a post-processing algorithm based on an amplitude analysis to enhance depth estimation solutions from potential field data. The central premises of the algorithm are twofold. First, automated depth estimation produces reliable result in selected windows but many false solutions are also introduced by the presence of noise and partial anomalies. Secondly, the reliable solutions have relatively much stronger source strengths. The enhancement is therefore achieved by statistically determining the threshold for the source strength so that solutions whose strengths are above it are retained as depth estimation result for further analysis. We have used extended Euler deconvolution to illustrate the proposed approach and present traditional Euler deconvolution on a field example.

Overall, the approach provides a much needed addition to depth estimation for magnetic basement mapping to reduce the number of false solutions. The use of multiple window sizes ensures that the solutions are not biased by pre-selected window size; and that the use of amplitude analysis brings back the missing amplitude information that is essential for any successful depth estimation problem. Utilizing amplitude in depth estimation algorithms now provides a basis for more quantitative work in fast and accurate interpretation of large-scale potential-field data sets.

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Figure 3: The histogram of the log of relative source strengths following a log-normal distribution. The cut-off between solutions caused by noise, such as spray, is chosen at one standard deviation from the mean. The result is fewer solutions that are more reliable and caused by basement contacts rather than noise.

Figure 4: The extended Euler locations after amplitude analysis are shown by white dots, the susceptibility is shown by the color bar, and the basement relief as shown by the contours. The results are either dominated by the relief, or by the susceptibility depending on the amplitude of the signal for both. For the solutions dominated by the susceptibility, the depths do not match the basement. The remaining results correspond well with the depth to basement. All of the results lie either on a change in susceptibility or change in relief.
REFERENCES
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