Hybrid optimization for lithologic inversion and time-lapse monitoring using a binary formulation

Richard A. Krahenbuhl¹ and Yaoguo Li¹

ABSTRACT

We have developed a hybrid optimization algorithm for binary inversion of geophysical data associated with lithologic inversion and time-lapse monitoring. The binary condition is designed to invert geophysical data for well-defined physical properties within discrete lithologic units, such as a salt body within a sedimentary host, or temporal changes in physical property associated with dynamic processes, such as the density and conductivity change in an oil and gas reservoir or groundwater aquifer. The solution of such inverse problems with discrete model values requires specialized optimization algorithms. To meet this need, we develop a hybrid optimization algorithm by combining a genetic algorithm with quenched simulated annealing. The former allows for easy incorporation of prior geologic information and rapid buildup of large-scale model features, whereas the latter guides genetic algorithm to faster evolution by rapidly adjusting the finer model features. In examining the performance of the hybrid algorithm, we note its superior performance. Our investigations of the algorithm’s capability with a 3D gravity inversion for the SEG/EAGE salt model and a time-lapse gravity data set from an aquifer storage and recovery process reveal its benefits.

INTRODUCTION

There have been many exciting developments in gravity inversion in recent years, but two general methodologies dominate the landscape of gravity inversion when applied to lithologic and time-lapse monitoring problems. The first methodology is interface inversion. Interface inversion methods seek to recover an interface that separates well-defined lithologic or time-varying units of interest through their discrete density contrasts. For example, in the time-lapse gravity problem, the methods may seek to recover a surface representing a 3D flow front as water is injected for enhanced oil recovery (EOR) during production. For lithologic inverse problems such as salt imaging, interface inversion methods typically assume a simple topology for the causative body and known density contrast and construct the base of the salt (e.g., Jorgensen and Kisabeth, 2000; Cheng, 2003). In the lithologic inversion problem, the methods have the advantage that they directly input a known time-varying density change from pore-fluid replacement during EOR, such as brine injection forcing desired hydrocarbons toward the producing wells. With such well-defined density changes, interface inversion methods directly reconstruct the interfacing surface between oil and brine over time as the injected water sweeps through the reservoir.

For the lithologic example of salt imaging, the interface methods have the advantage that they directly input the known density contrast at each depth and provide a direct image of the base of salt. However, the drawbacks of the method are that the assumed simple topology creates difficulties when regional field or small-scale residuals resulting from shallow sources are not completely removed during data processing to extract the anomalies of interest to be inverted. The inconsistency between the assumed model and data can lead to large errors or even failure of inversion. In addition, the problem is nonlinear and can be more difficult computationally.

Methods in the second category are generalized density inversions that seek to construct density contrast distribution as a function of spatial position. For the equivalent salt body problem, generalized density inversion methods often have the ability to incorporate prior geologic information and then image the base of salt by the transition in density contrast (Li, 2001). The generalized density inversions have the flexibility of handling multiple anomalies, highly complex shapes, and the solution is easier to obtain because the relationship between observations and density contrast is linear. However, as currently formulated, these methods are not well suited for cases with complex density contrast. For instance, they typically produce poor (if any) resolution near the nil zone in salt imaging. The difficulty arises primarily because of the existence of annihilators in the causative bodies. In such cases, density inversion methods allowing continuous density values (e.g., Li and Oldenburg, 1998) will in general
produce a model that has little resemblance to the true structure. The continuous change between general bounds does not impose a constraint that is as strong as we would like. As a result, the data are satisfied by intermediate density values and recovered distributions that only image a portion of the causative body.

To overcome difficulties associated with both methods, several authors use a binary formulation. Binary inversion belongs to a class of discrete inversion methods called lithologic inversions; they seek to reconstruct distinct lithologic units with specific physical properties within. When the assumption of only two lithologic units is adopted, it leads to the simplest case of a binary formulation. Thus, any binary inversion naturally expands its application to lithologic and time-lapse inverse problems.

The same difficulties associated with standard inversions can be overcome by incorporating prior information appropriate for the lithology or dynamic processes to restrict the class of admissible models. Binary inversion imposes the condition that the physical property must be the discrete values appropriate for the geologic or time-lapse problem. For example, in salt imaging, density contrast is restricted to being zero or a known value at a given location. Camacho et al. (2000) invert gravity data for a compact body with a constant density by growing the volume from an initial guess. Guillem et al. (2004) apply a constrained 3D cell-based gravity inversion to the Broken Hill Formation that seeks to modify the boundary or frontier cells between well-defined geologic units. Krahenbuhl and Li (2006) develop a binary inversion for imaging salt bodies using the Tikhonov formulation.

These binary methods are equally applicable to time-lapse inverse problems that monitor temporal changes in physical property associated with dynamic processes such as fluid contact movement (Krahenbuhl and Li, 2008) and aquifer storage and recovery (Davis et al., 2008). More recently, Silva Dias et al. (2008) develop a similar approach to salt imaging based on discrete density contrasts. The binary methods enable one to incorporate the physical property values appropriate to the geologic problems while providing a sharp boundary between different lithologic units in the subsurface. These are two strengths of the interface inversion.

The difficulty of the binary formulation, however, lies in the discrete nature of the physical property. Because the solution variables can only take on discrete values, derivative-based minimization techniques are no longer applicable as a solver for this category of highly constrained inverse problem. Many authors have developed different approaches to this problem. For example, Litman et al. (1998) apply the level set method in 2D obstacle reconstruction. Krahenbuhl and Li (2002, 2006) use a genetic algorithm (GA) to solve the optimization in binary gravity inversion. The appeal of GA is that it is intuitive and easy to implement. Consequently, it is expected to gain wider use in this class of geophysical inverse problems. However, a major impediment of GA is its computational cost. It is within this context that we seek to develop a hybrid optimization that improves upon the computational efficiency of the GA.

To develop the technique, we first briefly review the methodology of binary inversion to provide the background for our development of the optimization technique. We then present a hybrid optimization algorithm developed for efficient solution of binary inverse problems. We compare performance of the hybrid algorithm with that of stand-alone genetic algorithm. Last, we demonstrate the feasibility of binary inversion along with the hybrid optimization method in 3D inversion by applying them to the SEG/EAGE 3D salt model of Aminzadeh et al. (1997) and time-lapse gravity data from an aquifer storage and recovery (ASR) problem in Leyden, Colorado.

**BINARY FORMULATION**

Krahenbuhl and Li (2006) have formulated binary inversion by using explicitly the Tikhonov regularization approach (Tikhonov and Arsenin, 1977). The formulation was developed for the case of salt imaging at the presence of density reversal, but it is generally applicable. The problem then becomes one of minimizing an objective function subject to restricting model parameters to attain only one of two values at each location. The objective function consists of the weighted sum of the model objective function $\phi_{m}$ and data misfit $\phi_{d}$.

The minimization problem is then formulated as

\[
\text{minimize} \quad \phi = \phi_{m}(\rho) + \beta \phi_{d}(\tau),
\]

subject to \( \rho \in \{0,\Delta \rho(r)\} \),

(1)

where $\phi_{d}$ is formulated as the $\chi^{2}$ measure of our data misfit (Pearson, 1900; Fisher, 1924; Parker, 1977) and $\phi_{m}$ limits the solution of admissible models to those that are structurally simple (Li and Oldenburg, 1996).

Although discrete physical property values can take on different values in different cases, a general representation is to work with a binary model $\tau$ and scale it by expected density contrast at location $x,y,z:

\[
\tau(r) \in \{0,1\},
\]

\[
\rho(r) = \tau \Delta \rho(r).
\]

At a given location, a value of zero in the model $\tau$ indicates a zero density contrast, whereas a value of one corresponds to the expected salt density contrast at that location. When applied to the 4D gravity problem, a value of zero in the model $\tau$ represents regions of zero density change, such as background geology or unproduced reservoir volume, but a value of one corresponds to change in density over time associated with dynamic processes. The minimization problem is then expressed in $\tau(r)$, where $r$ is the location within the model, and we can work with zero and one for the minimization problem. The actual density contrast value associated with lithology or dynamic processes is incorporated into the forward modeling of predicted data during the inversion.

The minimization problem defined by equation 1 has a deceptive simple appearance, but its solution is not trivial. The difficulty lies in the discrete nature of the density contrast. Because the variable can only take on two values, zero or one, derivative-based minimization techniques no longer apply. There are several alternative methods for carrying out the minimization. One obvious technique is mixed integer programming (e.g., Floudas, 1995; Pardalos and Re-sende, 2002) because the variable to be recovered can only assume a value of zero or one. However, solving an integer-programming problem on a realistically sized 3D inversion is computationally prohibitive, and the cost for obtaining such an exact solution may outweigh the gains. An alternative approach is to utilize methods that seek a near-optimal solution to the minimization problem and demand far less computational cost. Examples include controlled random search techniques such as GA or simulated annealing (SA). Both methods are well suited for derivative-free minimization, which is the problem faced with binary inversion. In addition, GA
and SA can be implemented with relative ease compared to an integer programming solution and can yield solutions that are sufficiently accurate with much less computation.

Solving the binary inversion by GA offers the ability to easily incorporate density information and provide sharp boundaries for the subsurface while retaining the flexibility and linearity of density inversions. However, applying stand-alone GA to binary inversion for a large number of parameters still proves computationally expensive, precluding application to many practical problems. A similar conclusion is made by Zhang et al. (2004) who apply GA to inversion of gravity data in central Taiwan for 3D crustal structure. To better illustrate the application of GA as well as SA as solvers for binary inversion, we present inversion results for stand-alone GA and discuss the limitations of the two methods; then we introduce the hybrid optimization solver as an alternative.

**GA AND SIMULATED ANNEALING**

GA is a programming tool designed for solving a variety of optimization problems. This stochastic search technique mimics natural biological evolution by imposing the principle of "survival of the fittest" on a population of individuals. The main objective is to recombine the individuals (different models), with the better-fit individuals having higher probabilities of reproduction, in order for the population to evolve to better solutions. From an inversion standpoint, GA attempts to evolve a population of models to a final solution by ranking individuals and combining desirable model features at each generation. Additional information on GA can be found in Goldberg (1989), Pal and Wang (1996), and Chambers (1995a, 1995b). Information on applying GA specific to geophysical inversion is also available in Sen and Stoffa (1995), Smith et al. (1992), Sambridge and Mosegaard (2002), Scales et al. (1992), and Zhang et al. (2004), to name a few. Detailed information on the components of the GA for binary gravity inversion is available in Krahnenbuhl and Li (2006).

There are two primary advantages of GA for binary inversion. First, GA is ideally suited for minimization with the binary variable/model \( \tau \) composed entirely of zeros and ones. Cells within a geophysical model translate directly to a series of alleles (genetic analogy) for combination and mutation and are more manageable as an array of zeros and ones. For example, initialization and mutation by GA are straightforward, where only values of zero and one are assigned to the models during the former and binary flips from one to zero or vice versa are performed by the latter. Lack of magnitude in model perturbation during these stages of GA make it more efficient than in application to continuous-variable minimization. For example, mutation with the binary problem merely consists of randomly choosing cells to perturb, and then flipping them from zero to one, or vice versa. With a continuous variable problem, mutation requires first randomly choosing which cells to flip (similar to the binary problem) and, second, what level of change in amplitude to perform on those selected parameters during the mutation. This second step is naturally eliminated through the binary condition.

The second advantage of GA is its ability to work with multiple models at one time through the formation of a population, which easily allows the incorporation of prior geologic information. The information may be in the form of models generated from other geophysical data. For example, the top of salt from prestack-depth-migrated seismic data can be incorporated into the initial population in gravity inversion for base of salt. Similarly, the expected front of fluid substitution in an EOR process can be incorporated into the initial population.

SA is another global search technique well suited for inversion with binary variables. SA formulation is designed to mimic the process of chemical annealing, where the final energy state of a crystal lattice is determined by its rate of cooling through the melting point. To achieve a lower energy state with highly ordered crystals, the material must be cooled slowly. When the material is cooled too rapidly, the lattice may not reach the lowest possible energy state. The analogy in an inversion for this latter case is premature convergence, where the final solution has missed the desired global minimum. For geophysical inversion, SA typically starts with a model at random and calculates the model's energy represented by its objective value (equation 1). Perturbations are then applied to the model and the new objective values are calculated at each iteration. If the new objective value decreases or remains the same, the model is accepted as an updated solution. If the objective value increases, the model is accepted by a thermally controlled probability function often referred to as the Metropolis criterion:

\[
P = \exp\left(-\frac{\Delta \phi}{T}\right),
\]

where \(\Delta \phi\) is the difference between objective values of the old and new models and \(T\) is a temperature parameter designed to decrease (or cool) over time.

For more information on SA, the reader is referred to Metropolis et al. (1953), Kirkpatrick et al. (1983), and Nulton and Salamon (1988). Applications of SA specific to geophysical inversion, as well as sources of information on SA, are available in Dosso and Oldenburg (1989), Sen and Stoffa (1995), Nagihara and Hall (2001), Scales et al. (1992), Sambridge and Mosegaard (2002), and Roy et al. (2005).

SA is also well suited for the binary constraint because perturbations to the geophysical model require a mere binary flip from one to zero, or vice versa. If a parameter is selected by the SA to be perturbed, only one change can occur. Magnitude of the change is irrelevant with the binary inverse formulation. Forward calculation of the gravity response is therefore rapid, with the contribution of the flipped cell subtracted from, or added to, predicted data based on the new value of zero or one. With a temperature-controlled cooling function, SA works as a global search technique. As a result, this places SA in a similar category with GA as a global optimization solver.

GA and SA do have some disadvantages. As described above, they generate improved solutions to the inverse problem by utilizing information about the objective function directly rather than using derivative information. Because of this, they are among a small list of techniques available for the binary inverse problem. Unfortunately, both are computationally expensive. The difficulty of working with GA for large inverse problems is the forward calculation of data at each generation. Because GA works with a large population of solutions simultaneously, it may require hundreds to thousands of forward calculations at each iteration. In addition, the efficiency problem is compounded for larger inverse problems because an increase in parameters typically requires a larger population of solutions and an increased number of generations. Similar conclusions are drawn by Zhang et al. (2004) who likewise apply genetic algorithm to the
Hybrid formulation using GA and QSA

The motivation behind our hybrid algorithm is to develop an efficient, although not necessarily optimal (Chunduru, 1996), technique for working with the binary inverse formulation. In addition, the method must be able to incorporate prior geologic information into the inversion. As described, GA and SA are capable of working with binary variable and can manage prior geologic information; however, each method by itself contains undesirable features that one may wish to improve upon.

We therefore propose a hybrid algorithm in the fashion of Porsani et al. (1993) by implementing a local search to improve the top-fit individual of a GA (global) population of solutions. For this, we opt to use QSA as the local search tool in the hybrid algorithm. QSA in its simplest form is simulated annealing without the Metropolis or any other stochastic criteria for uphill moves, such as presented in equation 4. Therefore, QSA only accepts downhill and lateral moves. In our implementation of the hybrid approach, we do not activate the local search component at every generation, as do Porsani et al. (1993). Rather, GA is provided several generations to evolve a population of solutions, and QSA is implemented intermittently. The frequency of QSA activation throughout the larger GA varies, based on properties of a problem — such as the number of parameters. Our goal in incorporating QSA, however, is to speed up the evolution of a large population of solutions, thereby providing a faster and more efficient means of solving the binary inverse problem. The equivalent of implementing such a hybrid approach is the application of a pure GA for binary inversion with a rapid mutation operator applied to one individual every couple of generations by QSA.

One major source of the hybrid’s increase in speed over GA is associated with QSA, and in particular results from decreased forward model calculations while evaluating the total objective function in equation 1. To illustrate, at every generation of GA, the problem requires forward modeling of all model parameters for each new offspring model. If the problem requires many parameters and a correspondingly large population, then the forward-modeling component of equation 1 becomes expensive for GA. In contrast, a QSA perturbation is performed on a single model and requires a mere binary flip of a single parameter from one to zero, or vice versa. As a result, the precalculated gravity contribution of the selected model cell is added or subtracted from the predicted data. In short, if the desired model discretization is, say, 30,000 cells, then the forward modeling component of equation 1 by QSA requires 1/30,000 the number of calculations per iteration than GA for the same model. Our implementation of the hybrid algorithm may incorporate thousands of QSA perturbations at a time; however, it does not perform this level of processing on the entire population of solutions as does GA.

The effects of the hybrid algorithm on model structure during binary inversion are twofold. First, GA effectively develops large structure in the model. Second, complementary to GA, QSA improves the top fit individual after a prescribed number of generations by adjusting the finer details of the solution. This is an observation similar to that presented by Cary and Chapman (1988). Because the highest fit individual in the GA population has a strong influence on the evolution of the entire population, the “evolutionary jumps” provided by QSA result in faster convergence of a large number of solutions.

Performance of hybrid algorithm

To illustrate the performance of the hybrid algorithm, we apply it to an example that has the characteristics of a difficult salt imaging
problem, i.e., complex density profile, irregular shape of anomaly source, and many parameters. A 2.5D model is extracted from a section through the SEG/EAGE salt model of Aminzadeh et al. (1997). Density of the surrounding sediment increases with depth while that of the salt body remains constant. As a result, the top portion of the salt body attains a positive density contrast, the bottom has a negative contrast, and a nil zone is present around 2000 m depth (Figure 1a). The same 2.5D section is presented in binary form in Figure 1b.

There are 5670 cells in the model and 41 noisy data with zero mean and 0.025 mGal standard deviation, simulated in a profile above the model. To carry out inversion, we incorporate the top of salt as prior information, along with the expected density contrast function. The data are then inverted to find the shape of the lower portion of the salt body. We carry out the inversion with the binary formulation by GA and the hybrid algorithm to compare performance.

A stand-alone GA is first implemented with a population size of 1000 individual models. Half of the GA population advances at each generation, while the other half is created through selection, crossover, and mutation. Regularization is chosen such that the final $\chi^2$ data misfit is equal to the number of data.

The final solution by GA is presented in Figure 2 as a representation of the final population, i.e., an average of the population. This form of representation is not to imply that the average of the 1000 individuals is an acceptable solution in and of itself; instead, the purpose is to illustrate the geologic features common to and the variance throughout the individual solutions. GA reaches its final solution at approximately 500 generations. At this stage, the highest fit and average objective values of the population have converged, as illustrated in Figure 3a. The feature of interest in the GA convergence plot, which summarizes GA’s performance, is the similarity between the fits of the top individual and average of the population. There are few differences between the solutions at each generation; therefore, evolution of the population takes 500 generations and a total CPU time of 5 hours on a 2.4-GHz PC.

We next implement the hybrid algorithm to illustrate its performance in comparison to the GA solution described above. For this problem, the GA/QSA hybrid is initialized with a population size of 100 individuals. All other GA parameters, such as selection, crossover, and mutation, are consistent with the previous GA example. QSA is incorporated into the inversion every five generations of GA, acting on the top-fit solution for 5000 perturbations. The goal in choosing frequency and number of QSA iterations is to allow GA time to evolve a population of solutions while rapidly mutating a single individual. We want to avoid long processing time because of GA evolution, or rapid solution of a single model by QSA. The hybrid optimization is implemented to balance the two; and although QSA frequency is problem dependent, we have found that every fifth generation performs well here.

Figure 2. Inversion result for the 2.5D problem using GA. The model contains 5670 cells. There is a density profile and a thick nil zone. The result presented here is the average of a GA population of 1000 individuals. Regions of solid black or white are features of sediment or salt, respectively, which all members of the final population share.
Convergence of the population of solutions by the hybrid algorithm is illustrated in Figure 3b, in comparison with that of stand-alone GA (Figure 3a). The population converges to similar solutions in 50 GA generations. This is in sharp contrast to the 500 generations for pure GA. The total CPU time is approximately 4 minutes on the same PC.

The hybrid algorithm reduces the need for a large GA population with large genetic diversity, and it reduces the number of generations required for convergence. GA tends to dominate in the buildup of the larger model features throughout the population, whereas QSA modifies the top-ranking individual by rapidly developing the finer details in the model. The improvement achieved with the top-ranking individual leads to faster evolution of the entire population through evolutionary jumps in the hybrid algorithm. This is apparent in the performance plot in Figure 3b, with evolutionary jumps occurring every five generations as a result of QSA.

Also, the hybrid algorithm achieves a slightly lower value of the total objective function (Figure 3) than does the stand-alone GA. This is expected because GA evolution depends, to a great extent, on the inefficiency of a low-frequency mutation operator for continued minimization at the later generations. Ultimately, GA should achieve similar minimization levels as the hybrid algorithm; however, the unappreciable improvements in data fit and model structure at this asymptotic stage of evolution do not justify the significant increase in processing time.

Figure 4 shows the evolution of the GA/QSA hybrid model solutions over time, leading to the final inversion result. The images are presented as an average of the population of 100 models at specified generations, similar to results presented in Figure 2 by the GA. By the fifteenth generation of the hybrid, the solutions are good representations of the true model.

The desired aspects of the GA and QSA algorithms are successfully captured by the hybrid algorithm, which is the motivation for developing this solution strategy. First, the population of solutions allows one to incorporate larger amounts of prior geologic information, such as previous inversions, top of salt, and density information. However, no previous inversions were placed in the population for the above simulation. Second, the final solution may be represented as geologic information shared by multiple inverse models. Figure 4 represents an average of 100 final solutions generated by the hybrid algorithm. The average, in itself, does not represent an actual inverse model solution but illustrates those parameters common or varying within the 100 binary models. This feature is not available for single QSA inversions. Last, the algorithm significantly decreases processing time over that of GA. For the salt example, the difference in total CPU time is 4 minutes for the hybrid versus 5 hours for GA.

The hybrid algorithm has been successfully applied to the 2.5D problem of salt imaging by binary inversion of gravity data. The method demonstrates its ability to balance speed with quantity of information adequately, providing an alternative solution strategy for binary inversion over GA or QSA.

### 3D Inversion Examples

So far, we have focused on 2D examples for illustration in our efforts to develop a robust, efficient hybrid optimization algorithm. However, the ultimate goal is to invert data in three dimensions. We now turn our attention to this goal.

In principle, any numerical algorithm for inversion developed in two dimensions can be applied in three dimensions. The challenges are implementation and computational cost. With pure GA, 3D binary inversion could be prohibitively expensive even for a moderately sized problem and therefore become unfeasible to use. The hybrid algorithm is a step toward overcoming this limitation. We demonstrate this by applying the binary inversion to two 3D problems. The first is derived by simplifying the SEG/EAGE salt model, and the second is an application to an actual time-lapse aquifer storage and recovery problem.

#### 3D Lithologic Model

We first demonstrate the hybrid solver by applying binary inversion to a 3D problem derived from the SEG/EAGE salt model. The model (Figure 5a) was developed as a velocity model for seismic studies. For purposes of gravity studies, we have converted the velocity model to a generic cell-based model and incorporated density information in place of velocity (Figure 5b). The density model contains 28,350 cells measuring $300 \times 300 \times 300$ m each. As with the 2D problem in the previous section, we incorporate a background sediment density profile increasing continuously with depth. Given such a profile, the density contrast reverses sign from positive to negative at a depth of approximately 2000 m. Correspondingly, surface gravity data contain components of positive and negative anomalies. To simulate the surface response of the density model, 441 gravity data are generated above the model. Gaussian noise with zero mean and standard deviation of 0.1 mGal is added to the data (Figure 6).

The size of the problem makes its solution by simple GA unfeasible. However, the hybrid algorithm has performed well, as we illustrate below. The hybrid algorithm is initialized by generating a starting population of 201 models, with each model composed of uncorrelated random zeros and ones (Figure 7). Next, the top of salt is incorporated into the population as prior information to guide the inversion (Figure 8). In general, the top of salt may be forced to remain constant throughout inversion or may be allowed to change as pre-
ferred. For this illustration, the top of salt is permitted to change by GA mutation and QSA. At each generation, GA crossover, the step of generating new models, is performed by cutting two parent models into 3D blocks of cells. These blocks each contain 18 cells, and there are 1575 blocks within each model (Figure 9a-d). Next, the blocks of cells are combined randomly to generate the next generation of solutions from each parent pair (Figure 9e and f). Mutation is incorporated into GA by allowing each cell within the offspring models to undergo a binary flip with a probability of 1/3000. This corresponds to an approximate 10-cell mutation for each offspring model. QSA is applied to the hybrid algorithm every 25 generations of the GA on the top-ranking model. At this stage, cells within the model are flipped randomly for 20,000 perturbations. Any binary flips that do not increase the objective value of the model in equation 1 are accepted.

The hybrid solutions converge in 300 generations with a processing time of approximately 45 minutes on a 2.4-GHz PC. Figure 10 illustrates the performance of the population of models at each generation of GA, with the highest-fit model (lowest objective function value) undergoing evolutionary jumps because of QSA every 25 generations. Consistent with the convergence plot for the 2D example (Figure 3b), the best-fit individual here changes little by GA between the QSA iterations. In contrast, the average of the population changes drastically during these intervals by GA evolution. An exception is noted between the 75th and 100th generations. The top model improves because of GA, similar to that of the rest of the population.

Results of the 3D inversion by hybrid optimization are presented in Figure 11. The top image shows the true 3D salt model; the bottom...
image is the model recovered by the hybrid algorithm. The top of salt, while incorporated as a starting guide, is not held constant; therefore, slight changes to the upper portion of the salt body are observed. Locations of structure beneath the top of salt, as well as depths to the base of salt, correlate closely with the true model. Individual cells throughout the model region, which indicate isolated cells of salt, are attempts to fit noise during inversion.

The GA/QSA hybrid algorithm has been successfully applied to binary inversion of the full 3D SEG/EAGE salt model. Results show that the formulation has adequately resolved structure beneath the top of salt, generating a reasonable solution for the 3D lithologic problem.

**Time-lapse gravity problem**

The final example demonstrates the application of the hybrid solver by applying binary inversion to a time-lapse aquifer storage and recovery (ASR) problem. The Leyden mine, located in the city of Arvada, Colorado, was an active dual-seam underground coal mine until it closed in 1958. The city began to use the abandoned mine site for underground water storage by injecting purified water into the artificial aquifer in 2003. The goal of the project was to store excess water runoff during the wet winter months and to recover the water as necessary during the water-strained summer months. The ASR system in Leyden has negligible impact on surface environment, minimal maintenance costs, and no pan evaporation loss, so the underground storage system provides an affordable means of managing the city’s water-supply needs over traditional surface reservoirs. A full description of the site’s geology, history of water injection, and gravity data collection and processing can be found in Davis et al. (2008). Here, we use the processed data to illustrate application of binary inversion and the hybrid algorithm applied to the time-lapse gravity data collected over the ASR site as a means of monitoring the subsurface dynamics.

The ASR site, along with an overlay of the mine workings from historical records and the contoured difference gravity data, are illustrated in Figure 12. The difference map represents the change in

---

**Figure 9.** Generic example of crossover of two binary models in three dimensions by GA. (a, b) The two parent models, each with 28,350 cells. All of the cells from (a) are white, and all the cells from (b) are black. (c, d) The parent models are divided into 1575 blocks, each block containing 18 cells. (e, f) The blocks of cells from each parent pair are recombined randomly to form two children models. Each child model has opposite combination of block assemblage as the other child. For the actual inverse problem, cells within each block will not necessarily have the same values throughout the block.

**Figure 10.** Performance plot of the 3D binary inversion problem. The black points are the objective values of the highest-fit model at each generation; the gray points are the average objective value of the hybrid population at each generation.

**Figure 11.** The true model, which is to be reconstructed by the binary inversion algorithm. (b) The constructed model by the GA/QSA hybrid with binary inversion.
surface gravity field as a result of water injection over 10 months. The time-lapse gravity data are also presented separately in Figure 13. For this example, we focus on the 4D gravity data over the southern portion of the site only.

The mine workings for the site are at an estimated depth of 300 m below the surface. The model region for the binary inversion was designed to extend 60 m above the mine surface, twice the calculated maximum extent of predicted rubble zones associated with collapse (Sherman, 2003). The 3D model based on predicted coal mine workings (Figure 14) was used as a reference model. The model region was discretized into 38,080 cells, with 28 cells in the easting, 34 in the northing, and 40 in depth. Each cell within the model region has dimensions of $75 \times 75 \times 5$ m. We use a calculated rubble zone porosity of 35%, which translates to 0.35 g/cm$^3$ density change over time as a result of water fill (Davis et al., 2008).

The hybrid algorithm is initialized by generating a starting population of 81 models, with each model composed of uncorrelated random zeros and ones. As before, we implement the GA/QSA hybrid optimization algorithm as the solver for this time-lapse gravity ASR inverse problem. At each generation, GA crossover is performed by cutting two parent models into blocks of cells. These blocks each contain seven cells in the easting, eight cells in the northing, and 10 cells in depth. Mutation is incorporated into the GA with a probability of 1/3000. This corresponds to an approximate 13-cell mutation for each offspring model. QSA is applied to the hybrid algorithm every 20 generations of the GA on the top-ranking model. At this stage, cells within the model are flipped randomly for 20,000 perturbations. Any binary flips that do not increase the objective value of the model in equation 1 are accepted.

The hybrid solutions converge by 500 generations (Figure 15), with evolutionary jumps apparent every 20th generation. Results of the 3D inversion are presented in Figure 16, and they resemble the predicted mine workings within the central region of the site at original mine depths. The increased vertical thickness of the predicted model in comparison with the original mine workings also confirms that multiple rubble zones are present in the region of study. These rubble zones are the result of mine collapse at the end stage of mining, in which the remaining support columns are mined just before exiting the mine. Finally, there is a large region to the north where no density change is apparent in the final solution, indicating no significant presence of injected water there.

The hybrid optimization algorithm has successfully demonstrated application for 3D binary inversion in lithologic and time-lapse monitoring problems. It has shown that we can recover base of salt in a 3D setting in the presence of density contrast reversal. Similarly, it has demonstrated recovery of a meaningful solution to the ASR system in Leyden, Colorado, by inverting the time-lapse gravity data collected over a 10-month monitoring period. Such solutions were not easily acquired previously in three dimensions using GA in conjunction with the binary inverse formulation. However, with the de-
cesses, such as the density and conductivity change in an oil and gas reservoir or groundwater aquifer. The hybrid algorithm combines GA with QSA. The former allows for easy incorporation of prior geologic information and rapid buildup of large-scale model features, whereas the latter guides the GA population to faster evolution by rapidly adjusting the finer model features. We have demonstrated the superior performance of the hybrid solver for binary inversion by applying it to the 3D lithologic problem of salt imaging and time-lapse gravity monitoring problem of an active underground water storage reservoir.

ACKNOWLEDGMENTS

This work was supported by the Gravity and Magnetics Research Consortium (GMRC) industry consortium at the Center for Gravity, Electrical, & Magnetic Studies (CGEM) at the Colorado School of Mines. The sponsoring companies are Anadarko, BGP, ConocoPhillips, Chevron, BP, Vale, and CVRD. We thank Kris Davis for his diligent work in acquiring and processing the 4D gravity data over the Leyden ASR site.

REFERENCES

Aminzadeh, E., J. Brac, and T. Kunz, 1997, 3D salt and overthrust models, SEG/EAGE 3D modeling series, No.1: SEG.


Davis, K., Y. Li, and M. Batzle, 2008, Time-lapse gravity monitoring: A systematic 4D approach with application to aquifer storage and recovery: Geophysics, 73, no. 6, WA61–WA69.


Li, Y., and D. W. Oldenburg, 1996, 3-D Inversion of magnetic data: Geophysics, 61, 394–408.
