Comprehensive approaches to the inversion of magnetic data with strong remanent magnetization
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Summary

3D inversion of magnetic data to recover a distribution of magnetic susceptibility has been successfully used for mineral exploration in the last decade. However, the unknown direction of total magnetization has limited the use of this technique when strong remanence is present. In this paper, we propose a comprehensive methodology for solving this problem by examining a suite of approaches of practical utility. We illustrate these methods with a set of high-resolution aeromagnetic data acquired for diamond exploration in the Canadian Arctic.

Introduction

Quantitative interpretation of magnetic data through inversion for general distribution of magnetic susceptibility has been playing an increasingly important role in mineral exploration in recent years. Such applications range from district-scale to deposit-scale problems. Most currently available algorithms require the knowledge of magnetization direction, since it is an essential piece of information for carrying out the forward modeling (e.g., Li and Oldenburg, 1996; Pilkington, 1997). In most cases, one can simply assume that there is no remanent magnetization and the self-demagnetization effect can be neglected. Consequently, the direction of magnetization is assumed to be the same as the current inducing field direction. This is a valid assumption in a majority of the cases, as evidenced by many successful applications.

However, there are well-documented cases in which such an assumption is inadequate due to the presence of remanent magnetization. The total magnetization can be rotated away from the inducing field direction if the remanent magnetization is strong and not aligned with the inducing field. Without prior knowledge of the direction of resultant total magnetization, current algorithms become ineffective. This difficulty has limited the use of these inversion algorithms. For example, kimberlite pipes or dykes encountered in diamond exploration often have strong and nearly reversed remanent magnetization. As a result, magnetic inversion has not always been effective in generating interpretable 3D images of the source.

To address this issue, we propose a two-pronged approach. The first is to estimate the direction of total magnetization and supply it to the inversion algorithm, assuming that the magnetization direction does not vary greatly within the target region. There are a number of approaches for estimating magnetization direction. We examine their effectiveness in this paper. Alternatively, one might accept the fact that a single direction cannot be estimated for various reasons and opt to directly invert a quantity that is calculated from magnetic data but is independent of the magnetization direction.

In this paper, we use a field data set from diamond exploration as an example and investigate the issues related to the proposed approaches and provide guidelines that may be of help for field applications. In the following, we first review the essentials of these two methods and then apply them to a field data set as an illustration.

Estimation of total magnetization direction

Given the importance of the magnetization direction in the interpretation of magnetic data, it is not surprising that many authors have published on this subject. We present three methods here. The first two methods directly explore the relation between the anomaly and the magnetization direction and computes the magnetization direction from the data. The third method seeks to identify the properties possessed by the magnetic anomaly when it is reduced to the pole and identifies the magnetization direction by searching through a sequence of trial RTP’s using assumed direction. In both categories, we assume the magnetization direction does not vary drastically.

Helbig’s moment method


\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \nabla \times (x B_x (x,y))dx dy = -2 \pi m_x \]
\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \nabla \times (y B_y (x,y))dx dy = -2 \pi m_y \]
\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \nabla \times (z B_z (x,y))dx dy = -2 \pi m_z \]

where \( B_x \) and \( B_y \) are respectively the \( x \)-, \( y \)-, and \( z \)-component of the magnetic anomaly, and \( m_x, m_y, m_z \), and
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\( m_x, m_y, m_z \) are the three components of the magnetic moment of the source. Once the magnetic moment is estimated, it can be used to calculate the inclination and declination of the magnetization assuming they are constant within the source body.

Two scenarios arise in practical applications. First, we usually have only the total-field anomaly, and we need to first calculate the three components from the total-field anomaly by the corresponding wavenumber domain operators. Difficulties may arise when the data are acquired in low magnetic latitudes since the conversion involves a half reduction to the pole. Therefore, additional efforts are required near the magnetic equator. Alternatively, vector magnetic surveys have become available recently and the observed three-component data can be used directly in the estimation.

**Multiscale-edge method** Haney and Li (2002) proposed a method for estimating the magnetization direction in 2D using multiscale edges of a magnetic anomaly derived through a continuous wavelet transform. The multiscale edges correspond to the trajectories of the extrema of the wavelet transform of the anomaly profile, and their positions in the x-z plane are dependent upon the magnetization direction (inclination) of the source. Tracking the multiscale edges allows one to determine the inclination of the magnetization in 2D sources.

Although the method was developed for 2D problems, it can be applied to 3D data sets provided that the anomalies are reasonably isolated. In such cases, we can perform integration in each of the two horizontal directions to simulate two 2D profiles. Integrating in the easting direction simulates a profile along a north-south traverse above a 2D source that strikes in the east-west direction. Applying the method to this profile produces the apparent inclination of the magnetization within the north-south section. Performing similar operations in the perpendicular direction yields the apparent inclination in the east-west section. The inclination and declination of magnetization in the original 3D source can then be reconstructed from these two apparent inclinations.

**Cross-correlation method** The reduction-to-pole (RTP) anomaly theoretically has the least asymmetry of all magnetic anomalies produced by a given causative body. It follows that the vertical derivative of RTP anomaly is therefore also least asymmetrical. It has also been shown that the total gradient (amplitude of the gradient vector in 3D) of the RTP anomaly is the envelope of vertical derivative of the anomaly produced under arbitrary inducing-field and magnetization directions (Haney et al. 2003). The envelope, by definition, is the most symmetric form. Utilizing these properties, Dannemiller and Li (2004) develop a method for estimating magnetization by examining the symmetry of various RTP fields.

The method searches for the particular magnetization direction that achieves the maximum symmetry in the resultant RTP field. To measure the symmetry independently, we compute the cross-correlation between the vertical derivative and total gradient of the RTP anomaly that is calculated using an assumed magnetization direction. These two quantities should achieve the maximum correlation when the correct direction is used in RTP calculation. The estimation process becomes one of finding the inclination and declination that maximizes the cross-correlation over a grid of such trial directions.

The key operation of the method is the RTP process. Consequently, we will encounter difficulties at low magnetic latitude as before. Fortunately, several stable RTP methods are available for low latitudes.

We remark that there is a trade-off between the accuracy of estimated inclination and the accuracy of estimated declination. The accuracy of inclination improves as it approaches -90° or 90°, but the accuracy of the corresponding declination decreases. However, this does not pose a major problem since the influence of the declination becomes less important at high latitudes.

**Inversion of direction insensitive data**

Amplitude of anomalous magnetic field, or the total gradient of the magnetic anomaly vector, is independent of the magnetization direction in 2D problems (Nabighian, 1972). Although such a property does not extend exactly to 3D problems, both quantities are only weakly directional dependent. This is especially true for total gradients when the anomaly has been converted to the vertical component by a half reduction to the pole. This property provides the opportunity for direct inversion of the anomaly amplitude or total gradient to recover the magnitude of magnetization without knowing its direction.

Shearer and Li (2004) develop such an algorithm by formulating a generalized inversion using Tikhonov regularization and imposing a positivity constraint on the amplitude of magnetization. The algorithm starts by calculating, for example, the amplitude of the anomalous magnetic field from the observed total-field anomaly. It then treats the amplitude as the input data and recovers the distribution of magnetization as a function of 3D position in the subsurface. One advantage of the approach is that it is not limited to a single anomaly nor does it require that adjacent anomalies have the same magnetization direction. Therefore the approach is potentially applicable to a wide range of problems where the source distribution is more complicated.

**Application to field data**

Figure 1 shows a set of high-resolution aeromagnetic data over a kimberlite dyke acquired by TeckCominco and
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Diamonds North over Victoria Island in Northwest Territory, Canada. The inducing field has an inclination of 86.67° and declination of 26.33°. Given the high magnetic latitude and dominant negative anomaly, it is clear that the kimberlite has strong remanence and the total magnetization is nearly in opposite direction from the inducing field. For comparison, Figure 2 shows the amplitude of the anomalous magnetic field obtained by converting the total-field anomaly into three orthogonal components.

![Figure 1](image1.png)

**Figure 1.** Total-field magnetic anomaly (nT) over a kimberlite dyke. The inducing field is in the direction of I=86.67° and D=26.33°. Judging from the negative anomaly in the center, the presence of strong remanent magnetization is apparent.

![Figure 2](image2.png)

**Figure 2.** The amplitude (nT) of the anomalous-field vector computed from the total-field anomaly shown in Fig. 1.

We invert the data using both approaches: we first estimate the total magnetization direction from the total-field anomaly in Figure 1 and invert it accordingly. Next, we invert the amplitude data in Figure 2 directly. Both results will be compared with that from methods currently used in the mineral industry (e.g., Paine et al., 2001).

![Figure 3](image3.png)

**Figure 3.** Two profiles simulated from the data map in Fig. 1 by performing integrating in north-south direction (top) and east-west direction (bottom).

<table>
<thead>
<tr>
<th>Method</th>
<th>Inclination (°)</th>
<th>Declination (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helbig’s</td>
<td>-84.7</td>
<td>70.0</td>
</tr>
<tr>
<td>Multiscale edges</td>
<td>-89.3</td>
<td>1.8</td>
</tr>
<tr>
<td>Cross-correlation</td>
<td>-87.4</td>
<td>26.0</td>
</tr>
</tbody>
</table>

**Table 1.** Magnetization direction estimated using three different methods.

As an illustration, we present the result from the first approach here. To estimate the magnetization direction, we have applied the three methods reviewed earlier. For Helbig’s method, we calculate the three-component anomaly in the wavenumber domain. For the wavelet approach, we use the simulated profiles shown in Figure 3. The results of estimation are listed in Table 1 for comparison. We note that the estimated values for inclination are similar but the declination varies greatly. This is to be expected given the inclination is close to -90°. When used in an inversion, the error in declination does not affect the final result either.

With the estimated magnetization direction, we apply the 3D inversion algorithm developed by Li and Oldenburg (1996). The mesh uniformly discretizes the model region directly below the data area and down to a depth of 200 m. The cells have a width of 10 m in both horizontal directions and a thickness of 5 m. Outside this core region, the mesh is extended outward and downward by cells of increasing sizes in accordance with usual practice.

The recovered effective susceptibility using the direction estimated from multiscale edges is shown in one cross-section at 300-m north and one plan section at a depth of 50 m in Figure 4. The inversion effectively images a compact
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magnetic body. It has a strike in northwest direction and a strike length of approximately 250 m and is located at a depth of 50 m to the center. This result is consistent with the presence of a kimberlite dyke.

Figure 4. Recovered effective susceptibility (10^-3 SI) by inverting the total-field magnetic anomaly in Figure 1. The inversion uses the magnetization direction estimated by the multiscale-edge method. The top panel is a cross-section at 300-m north and the lower panel is a plan section at a depth of 50 m. The inversion clearly images the presence of a kimberlite dyke.

Discussion

We have presented a comprehensive method to the problem of inverting magnetic data in the presence of strong remanent magnetization that alters the direction of the total magnetization. The method consists of two approaches. The first approach directly addresses the issue of unknown magnetization direction and estimates it using several existing and newly developed algorithms. The data are then inverted using existing magnetic inversion algorithms. The second approach circumvents the need for reliable knowledge of magnetization direction and, instead, inverts directly the amplitude of the anomalous field (or the total gradient of the observed data) to recover the magnitude of the magnetization. Thus, there is now set of tools at our disposal for interpreting magnetic data in the presence of strong remanent magnetization.

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