Rapid gravity and gravity gradiometry terrain corrections via an adaptive quadtree mesh discretization

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Abstract. We present a method for modelling the terrain response of gravity gradiometry surveys utilising an adaptive quadtree mesh discretization. The data- and terrain-dependent method is tailored to provide rapid and accurate terrain corrections for draped and barometric airborne surveys. The surface used in the modelling of the terrain effect for each datum is discretized automatically to the largest cell size that will yield the desired accuracy, resulting in much faster modelling than full-resolution calculations. The largest cell sizes within the model occur in areas of minimal terrain variation and at large distances away from the datum location. We show synthetic and field examples for proof of concept. In the presented field example, the adaptive quadtree method reduces the computational cost by performing 351 times fewer calculations than the full model would require while retaining an accuracy of one Eötvös for the gradient data. The method is also used for the terrain correction of the gravity field and performed 310 times faster compared with a calculation of the full digital elevation model.

Key words: gravity, gravity gradiometry, processing, quadtree, terrain correction.

Introduction

The effect of terrain in gravity gradiometry surveys for petroleum and mining applications is often the predominant signal measured, because the field decays as distance-cubed (Chen and Macnae, 1997) and the terrain variation is the closest source. Therefore accurate terrain corrections are critical to proper interpretation of the dataset. However, the accuracy of the terrain correction is most often a function of the resolution of the available digital elevation model (DEM). As a result, higher and higher resolution DEMs are acquired during gravity gradient surveys. While this results in more accurate corrections, these high-resolution DEMs have the unfortunate consequence of creating extremely high computational demands. Current methods of terrain correction often forward model the terrain effect with reasonable coarsening of terrain as distance from the observation location increases. A major portion of the terrain is calculated at full resolution. Such an approach can take days and sometimes weeks to complete for large-scale surveys. Much faster algorithms are needed if interpretation of data is required in the field or if results are simply needed quickly.

Terrain corrections for gravity data also require high resolution DEMs, especially for ground-based surveys. In addition, a much larger spatial extent of the DEM is required for gravity surveys as compared to gravity gradient as the field decays as distance-squared. Thus terrain from greater distances affects the calculation. While the minimum DEM extent and cell size requirements for ground-based gravity surveys are well defined (Hammer, 1939), airborne gravity terrain corrections are more troublesome and less defined (Hammer, 1974). Similar issues exist with gravity as with airborne gravity gradiometry surveys.

Although extremely high resolution DEMs are required in areas with large topographic relief, the resolution requirements can be relaxed as the distance from the observation point increases. Also, if the characteristics of the terrain change over the survey, the resolution requirements of the DEM may change as well. Rough or rugose terrain will require finer discretization to accurately represent the rapidly changing elevation. Furthermore, the required resolution is also a function of the flight height and acquisition filters (Reid, 1980; Kass and Li, 2008). Thus a data-adaptive discretization of the DEM can result in faster computations and smaller storage requirements while ensuring the desired accuracy.

In this paper, it is shown that the adaptive quadtree method is appropriate for data-dependent discretization of DEMs for terrain corrections. The method allows for user-defined error bounds and parallel computing capability, and reduces the number of required calculations by two orders of magnitude in the example. We start with the forward modelling calculation of the prismatic model and then discuss the discretization method. Synthetic and field examples are shown for illustration.

Forward model approach

The terrain effect in gravity gradiometry is calculated as the response due to the region of mass bounded above by the terrain and below by a plane that passes through the lowest elevation of the survey area. We discretize this region into vertical prisms, each with a constant density, and calculate their individual contributions. We have adopted a Cartesian coordinate system having its origin at the Earth’s surface, x-axis pointing towards grid north, y-axis pointing towards grid east, and z-axis pointing vertically downward. The forward model of the gravity gradient follows a similar construction as described in Li (2001). As gravitational potential satisfies Laplace’s equation, the potentials due to all sources superimpose at each observation point and thus the contribution of all prisms to the gravity gradient measured at a point is the sum of their individual responses. For a
source at location \( \mathbf{r} \) with density \( \rho \), the gravity gradient at an observation point (denoted by vector \( \mathbf{r}_a \)) is a tensor \( \mathbf{T} \) with individual components represented by

\[
T_{ki} = \gamma \rho \frac{\partial^2 \mathbf{r}}{\partial \mathbf{r}^2} \frac{M}{|\mathbf{r} - \mathbf{r}_a|} dV \quad \text{for } k, l = x, y, z, (1)
\]

where \( V \) is the volume of the source, \( \gamma \) is the gravitational constant, and \( k \) and \( l \) represent the three orthogonal directions described above. The integral for a rectangular prismatic volume follows the magnetic derivation by Sharma (1966) or Zhang et al. (2000), with treatment to avoid singularities at observation locations coplanar with the faces of the prisms. As the equations are lengthy and not required for understanding of the method, we direct the reader to Sharma (1966) for a detailed discussion of the integral solution. For source-free regions, this tensor is symmetric and traceless, and commonly is written in the form:

\[
\mathbf{T} = \begin{bmatrix}
T_{xx} & T_{xy} & T_{xz} \\
T_{yx} & T_{yy} & T_{yz} \\
T_{zx} & T_{zy} & T_{zz}
\end{bmatrix}.
\]

(2)

We can write the expression for the gravity gradient at a datum point, \( \mathbf{d}_i \), as

\[
\mathbf{d}_i \equiv T_{ki}(\mathbf{r}_a) = \sum_{j=1}^{M} \gamma \rho_j \left( \frac{\partial^2 \mathbf{r}}{\partial \mathbf{r}^2} \frac{1}{|\mathbf{r} - \mathbf{r}_a|} dV \right) = \sum_{j=1}^{M} \rho_j g_{ij}.
\]

(3)

In this construction, we are summing the response due to \( M \) prisms, \( \Delta V_j \), of constant density \( \rho_j \), having vertical extents from the surface to the bottom plane. The forward model of the gravity field follows a similar approach. The field due to a volume source is represented as

\[
g_e(\mathbf{r}_a) = \gamma \rho \int \left( \frac{z - z_0}{|\mathbf{r} - \mathbf{r}_a|} dV, (4)
\]

leading to the gravity field at one data point \( \mathbf{d}_i \) due to the sum of responses:

\[
\mathbf{d}_i = \sum_{j=1}^{M} \gamma \rho_j \left( \frac{z - z_0}{|\mathbf{r} - \mathbf{r}_a|} dV \right) = \sum_{j=1}^{M} \rho_j g_{ij}.
\]

(5)

It is important to note that the matrix (\( \mathbf{G} \) or \( \mathbf{S} \)) can be quite large. However, the forward modelling algorithm does not require the formation and storage of the complete matrix at one time, but rather only the computation for one datum at a time. Furthermore, the discretization of prisms varies from datum to datum and thus the value of \( M \) and lengths of the rows of the matrix are constantly changing. This is fundamental to providing rapid calculations with minimal storage. It is also important to note that the varying length of \( M \) and the principle of superposition are the key properties allowing for parallelisation of the computation using multiple threads.

Algorithm

Traditional approaches to terrain correction rely on prescribed coarsening of the elementary approximation to the terrain as a function of horizontal distance from the observation location. To extend this approach and remove the change of prescriptions that may be sub-optimal, we propose an adaptive coarsening of the system based on a quadtree mesh discretization of terrain for each observation location. The discretization algorithm considers both distance and terrain relief.

Quadtrees are a hierarchical representation of information. They are primarily used in data structures and data mining (Harrington, 2000) to relate details with general headings or information in order to save computation time or storage space. Quadtree mesh discretization has been primarily used in geophysics for remote sensing applications for the storage of images (e.g., Gerstner, 1999). The image decreases in size by assembling similar pixels into groups, yet areas of interest or rapid change retain the original pixels to maintain high resolution. Recently, quadtrees have also found application in 2D DC resistivity inversion (Eso and Oldenburg, 2007). DC resistivity modelling incorporates a similar idea with smaller cells grouped into larger cells based on distances between source or observation locations and mesh elements. The quadtree method is particularly useful in large scale problems by minimising the number of required model parameters (Frey and Marechal, 1998; Ascher and Haber, 2001). The method discretizes the model more finely at locations of desired high resolution areas and discretizes coarsely where low resolution is permitted. This type of local grid refinement is used in our application of terrain effect calculation. We adopt an algorithm that starts with a low resolution discretization of the terrain and iteratively splits parameters to increase resolution up to the original resolution of the digital elevation model based on the gravity or gravity gradient response. This is done for each observation location.

The mesh begins as large prismatic cells (i.e., a \( 32 \times 32 \) mesh) that have grouped terrain values of the smallest cell size (e.g., the DEM grid spacing) within them. The forward-model response of the large cell is calculated first using an average height. Next, the cell is split into quarters and new average heights are used to calculate the combined response of those cells. If the values differ by more than a defined local error level, the four cells are kept for further analysis. Otherwise, the large cell first modelled is used. This process continues iteratively until all responses are within the given error level or the resolution has reached the full resolution of the DEM.

In this application, we define error as the data difference between the full resolution and quadtree models. It should be noted that this defined error does not include the inherent errors in the DEM. Thus, the accuracy of the method is bounded by both the DEM resolution and the respective errors in the acquisition of the terrain. The error level for the quadtree algorithm is normalised by the number of cells in the model creating a local error level for each group of cells. Thus, the user-specified error level is the sum of the maximum local error of each model element. Hence, if a cell is split (into four smaller cells), the local error threshold for that cell is divided by four in order to preserve the upper bound for the grouped cells. For example, if the initial model is split into 256 cells, then the first local error level is the user-defined error divided by 256. If the initial cell is split into quadrants, the local error for those quadrants is \( 1/(256 \times 4) \) or \( 1/1024 \) of the user-specified error. The local error level is a termination mechanism for the quadtree mesh discretization. Although the error is an upper bound, in practice the true error is much smaller, as shown in the examples.

Selection of the initial discretization has no effect on final accuracy. However, the two end members where the discretization is close to the initial DEM interval and where the cell size is half the DEM (e.g., \( 2 \times 2 \)) should be avoided. We have empirically determined that a \( 32 \times 32 \) initial discretization is a good starting point in most cases.
Examples

Gaussian hill

We first use a synthetic example based on the Gaussian hill model, presented in Figure 1, to illustrate the accuracy of the method. The function used to generate the DEM is given by:

\[ T(x, y) = 300 - 300 \exp \left( -\frac{x^2}{300^2} - \frac{y^2}{300^2} \right). \]  

The hill extends from 0 m at ground level to 300 m above the ground at its peak and has a positive density contrast of 2.65 g/cc. The total number of observation locations was 625. We first calculated the gravity gradient responses of this terrain at 350 m above ground level for each component and allowed a 1-Eötvös error. The starting cell size is 1600 m by 1600 m based on the longest radial wavelength. We show the adaptive discretization of a Gaussian hill for a single observation location directly above the peak of the hill in Figure 1b used in gravity gradiometry calculation. The asymmetry seen in the discretization about the x and y axes is a result of the centre of the initial cells before quadtree-splitting not being coincident with the peak of the model. However, we do observe the appropriate axis of symmetry which is rotated 45 degrees above the positive abscissa and through the origin.

We compared the gradient response of the Gaussian hill calculated with our adaptive quadtree discretized DEM (Figure 2) to the full resolution calculation based on a regular grid. Although we allowed the algorithm as much as 1-Eötvös error, the maximum difference between the results from these two approaches was 0.17 Eötvös (Figure 3).

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Fig. 1. (a) 300 m high Gaussian hill with a density contrast of 2.65 g/cc used as a synthetic test dataset. (b) Automated adaptive quadtree discretization determined by the algorithm that achieves a minimum of 1-Eötvös accuracy 50 m immediately above the peak for all tensor components.
Fig. 2. Gravity gradient response for each component on a plane 50 m above the peak of the 300 m-high Gaussian hill model. Units are in Eötvös.
Fig. 3. Difference between the gravity gradient terrain effects calculated with the adaptive quadtree method versus a full-resolution surface-based method. The maximum error threshold between these two models was 1 Eötvös. Units are in 1/100 of an Eötvös (−1 to 1 Eötvös colour bar).
We examine the number of calculations performed in order to evaluate the efficiency of the adaptive algorithm. The full resolution model contains 729,088 cells, which means that there are 455,680,000 required calculations. Using the 1-Eötvös error level example, our method made 1,468,486 calculations, 310 times less than (or 0.3% of) the full model.

We also use this terrain to demonstrate the method for gravity terrain correction. Figure 4 is the model generated by the adaptive quadtree discretization for a single data location, similar to the gradiometry model. The accuracy was set to a maximum of 0.05 mGal. It took 129 times fewer calculations than the full model required (3,522,960 versus 455,680,000 calculations). The data generated from the model is shown in Figure 5. The difference between the full model and quadtree discretized model is similar to that of the gravity gradient data.

Field dataset

To demonstrate the efficiency of the algorithm using a realistic scenario, we calculated the terrain corrections for a draped airborne gravity gradient and gravity survey from an Airborne Gravity Gradiometry test site in Australia (Figure 6). There are 8379 observations locations within a 5000 m by 5000 m area. The terrain data is sampled at 10 m intervals in a 40,000 m by 40,000 m area. We first examine the gravity gradiometry response of the terrain, and then the vertical gravity field.

The gravity gradient terrain corrections used a density contrast of 2.67 g/cc and allowed a maximum of 1-Eötvös error while the gravity corrections used the same density contrast and 0.05 mGal error tolerance. Figure 7a shows the discretization of the DEM for one observation location. The immediate area surrounding the datum is presented in Figure 7b for clarity. The discretization
requiring the minimum cell width (e.g. the DEM interval) is asymmetric. This is due to the hill to the east of the datum, which requires finer discretization than the flatter terrain to the west. This exemplifies the utility of the method to take into account the response of the terrain, rather than having full dependency on the distance away from the datum. The gradiometry correction took 351 times fewer calculations, meaning there were ~0.3% of the calculations necessary to produce the corrections as compared to using full-resolution. Figure 8 shows the calculated components of the gravity gradient.

The terrain correction for an airborne survey of the vertical component of the gravity field was calculated in the same manner. Figure 9 shows the corrections in mGal, which have the expected features. These appear smoothed due to the observation elevation and the less decay rate than the gradiometry data. The full resolution model would require 133 862 979 411 calculations,
Fig. 8. Gravity gradient tensor terrain effect at the 8179 observation locations. The density used was 2.67 g/cc allowing a maximum of 1-Eötvös error. The quadtree method required 351 times fewer calculations than for the full resolution model.
whereas the quadtree discretization only needed 421,056,840 to be within the 0.05 mGal error tolerance. The correction took 318 times fewer calculations than for the full resolution model.

Conclusions
An efficient algorithm has been developed for rapid calculation of terrain effects in airborne gravity and gravity gradiometry surveys. The method employs quadtree mesh discretization to represent the DEM adaptively for each datum based on the distance from the observation point, local ruggedness of the terrain, and a user-specified error tolerance. The algorithm achieves a much higher computational efficiency than the traditional full calculation of the DEM, and allows for more accuracy than methods that use prescribed discretization based on distance alone. Depending on the complexity and relative relief of the terrain model, the adaptive quadtree correction method can speed up the calculation by two orders of magnitude over currently available algorithms. The new algorithm is well suited for application in modern airborne surveys that are characterised by a large number of observations, highly variable altitude, and accompanying high-resolution DEMs such as those generated by light detection and ranging (LiDAR) surveys. Thus, this algorithm provides a valuable and efficient alternative for calculating gravity and gravity gradient terrain corrections in modern airborne surveys.

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四分木メッシュ分割を用いた重力および重力勾配の高速地形補正

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要旨：四分木メッシュ分割によるグリッド値推定法を利用して、重力勾配調査における地形補正値のモデル化の方法を試みた。この手法は、対地高度一定もしくは高度一定のエリア内重力の調査結果を高速かつ正確に地形補正するために、基準面を用いた地形補正の SHORT Snap 分割を行う一方法である。この方法では、地形補正值モデルの都合面を、精度に悪影響を及ぼさない程度で最大のセルサイズに自動的に分割してモデルを作成する。その結果、高解像度のセル分割を用いた計算よりも、高速化に計算できる。モデル中のセルサイズは、地形変化が小さい基準面から離れた位置において最大になる。

本手法の検証のため、合成記録およびフィールドデータの例を示す。このフィールド例では、勾配データを1エトヴェスの精度で計算する場合、四分木分割法では、通常の全域モデル法に比較して351倍高速に計算でき、計算コストを節約できる。また、重力層の地形補正にこの手法を用いたところ、標高データ（DEM）を全て計算する場合と比較して310倍高速に計算することことができた。

キーワード：重力、重力勾配、データ処理、四分木、地形補正

最適4ジントリク 어자화를 이용한 중력 및 중력 변화율 탐사에서의 고속 지형보정

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요약：최적화된 4진트리 어자화 기법을 이용한 중력변화율 탐사의 저형 효과 계산 방법을 제시하고자 한다. 계산하고자 하는 방법은 항상최소의 자료처리를 위하여 저형 자료의 최적화된 빠르고 정확한 지형특수 계산법이다. 각 지점에서의 저형효과 계산에 이용되는 자료 정도 자료는 자동적으로 원하는 정밀도를 제공할 수 있는 최대 크기로 어자화되어 최대 해상도 자료를 이용하는 방법에 비하여 뚜른 계산이 가능하다. 이러한 최적화된 적자 크기는 각 지점에서의 겉사와 지표의 고도 변화를 고려하여 구성을된다. 새로운 접근 방법을 검증하기 위하여 수치모델링과 실험자료에 적용하였다. 현장 자료에 적용한 결과 최적 4진트리 기법은 최고 해상도 자료를 모두 이용한 방법과 비교하여 중력 변화율 자료에서 1EU(Eötvös unit)의 정밀도를 유지하면서 계산량은 1/351로 줄일 수 있었다. 또한, 중력탐사 결과의 지형보정에 이용한 결과 모든 DEM자료를 이용한 계산에 비하여 310배나 뚜른 계산이 가능하였다。

주어：중력, 중력 변화율, 자료처리, 4진트리, 지형보정

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